A Method of Using Quadrature Sampling to Measure Phase and Magnitude

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Introduction

The purpose of this paper is to describe a method for measuring the phase and magnitude of a sinusoidal signal by using four reference square waves of the same frequency, each offset from the previous one by 90 degrees. This mechanism is easily implemented for mid-audio frequencies, such as 5 KHz. What makes it useful is that the task of doing phase and magnitude measurements of RF signals can often be reduced to doing the same at a low IF frequency such as 5 KHz.

The Value of Quadrature Sampling

Figure 1 shows a graph of a sinusoidal signal fluctuating 2V above and below a DC offset of 0.2V. If we sample the signal at 90 degree intervals from an arbitrary starting point of 0 degrees, the first four samples will be as shown.

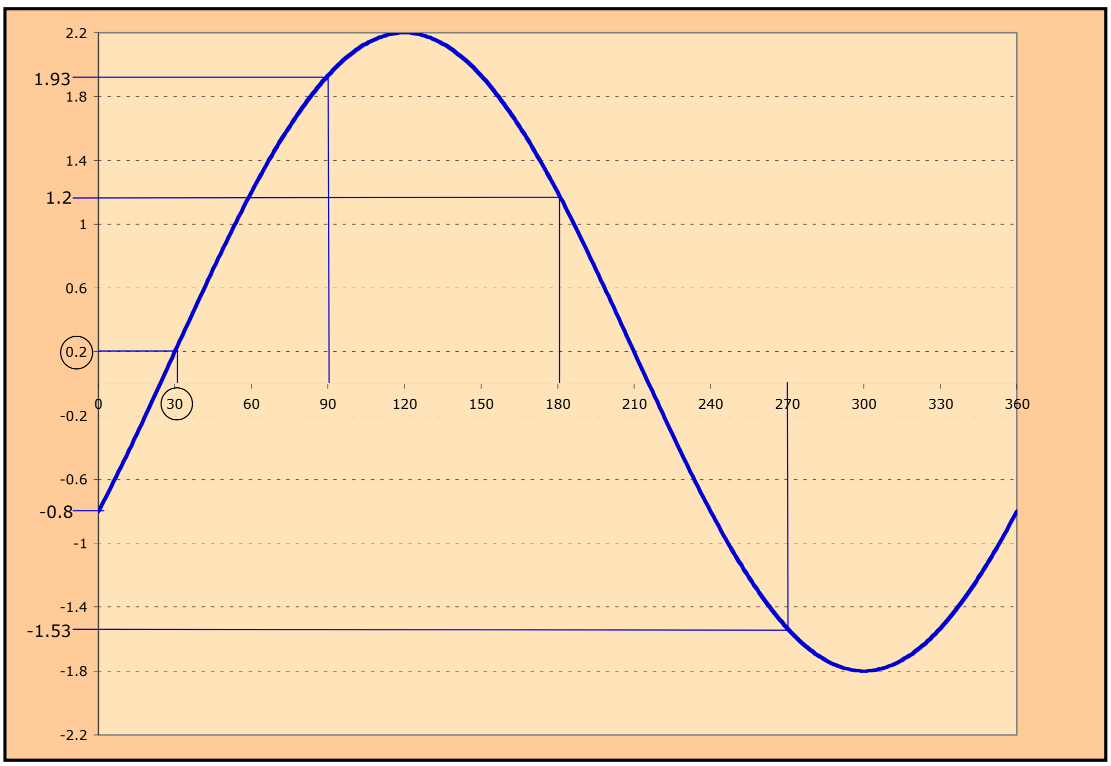

Figure 1—Graph of a sine wave of amplitude 2, with vertical offset 0.2 and phase delay of 30 degrees. It is the graph of 2*sin(x-30) + 0.2. The values of the function at 0, 90, 180 and 270 degrees are shown as -0.8, 1.93, 1.2 and -1.53 respectively.

This gives us two sets of measurements spaced 180 degrees apart (0/180 and 90/270) and two sets of measurements spaced 90 degrees apart (0/90 and 180/270). Let's look at what information this provides us. First, we know that the sines (or cosines) of two angles spaced 180 degrees apart will be negatives of each other, so their average will be zero. In this case, the values at 0 and 180 degrees average to (1.2 - 0.8)/2, or 0.2V. This indicates that the signal is shifted upward 0.2V, or in other words has a DC offset of 0.2V. The same result is obtained using the 180/270 measurements. In the real world, the two results would probably be somewhat different, so we could average the two to get a better estimate of the DC offset.

The real payoff comes from the measurements spaced 90 degrees apart, known as quadrature measurements and referred to as I and Q. If we subtract the DC offset from the 0 and 90 degree measurements, we will call the results I and Q, respectively. The magnitude of any perfect sine wave (i.e. its peak value) is determined as follows:

Peak Magnitude= Sqrt(I^2 + Q^2)

This is true of any two quadrature measurements—i.e. any two measurements spaced 90 degrees apart. It is related to the fact that the sine of an angle is equal to the cosine of the same angle increased by 90 degrees, and that the sum of sin^2 + cos^2 equals 1.

If we use the values at 0 and 90 degrees, corrected for the 0.2V DC offset, we get I=-1 and Q=1.73. The sum of the squares of these values is 3.993, whose square root is 1.998, very close to the true peak value of 2. The error is due to rounding errors in the measurements. We could use the values at 180 and 270 degrees and get the same result.

We can go one step further. The phase angle of a sine wave sin(x + p) is equal to p, which in turn is equal to the following:

θ=Arctan(I/Q) or Arctan(I/Q) ± 180

The Arctan produces a value from -90 to 90. For negative values, we can add 180 degrees and get another solution such that Tan(θ) =I/Q. For positive values we can subtract 180 degrees to get the second solution. We determine the appropriate solution by selecting the value θ such that sin(θ) has the same sign as I, and cos(θ) has the same sign as Q. If Q is zero, then θ is 90 degrees for positive I, and -90 degrees for negative I.

The value of θ we obtain by this method treats the I measurement as being taken at phase 0 and the Q taken at phase 90. In other words, it uses the time of the I measurement as the reference for establishing zero phase.

Using the 0 and 90 degree measurements as I and Q (after subtracting the offset), we get θ=Arctan(-1/1.73)= -30 degrees or 150 degrees. In order to make the sign of sin(θ) to be negative (to match I) and the sign of cos(θ) positive (to match Q), we conclude that θ=-30 degrees. This corresponds to the "-30" term in the equation graphed in Figure 1.

Using the 180 and 270 degree measurements, we would get θ=Arctan (1/-1.73) as before, but this time we would select 150 as the value for θ so that sin(θ) would be positive and cos(θ) would be negative, matching the signs of I and Q, respectively. But as noted above, the formula treats the I measurement as phase 0, whereas in this case our I measurement is taken at phase 180 degrees. So to use the same zero reference as before, we need to subtract 180 degrees from our θ value of 150, which produces -30 degrees. Again, in the real world the two results would differ somewhat, and we would get the best estimate of θ by averaging the two results.

In summary, if we have a pure sine wave and we can make measurements at 90 degree intervals for one complete cycle, we can determine the DC offset, peak magnitude and phase of the signal. Plus, we get two determinations of each value, which allows us to average the two to reduce the effects of noise and measurement error.

How to Accomplish Quadrature Sampling

In the general case, it is not easy to space our measurements exactly 90 degrees apart for a sine wave of arbitrary frequency. But if we have a hand in generating the original sine wave, the job can be much easier. If we can create four 5 KHz square waves phased at 90 degree intervals, we can use these signals to generate the RF test signal, and to control the timing of the measurements of the 5 KHz test signal we ultimately derive from the RF. The circuit for generating four phased square waves is shown in Figure 2.

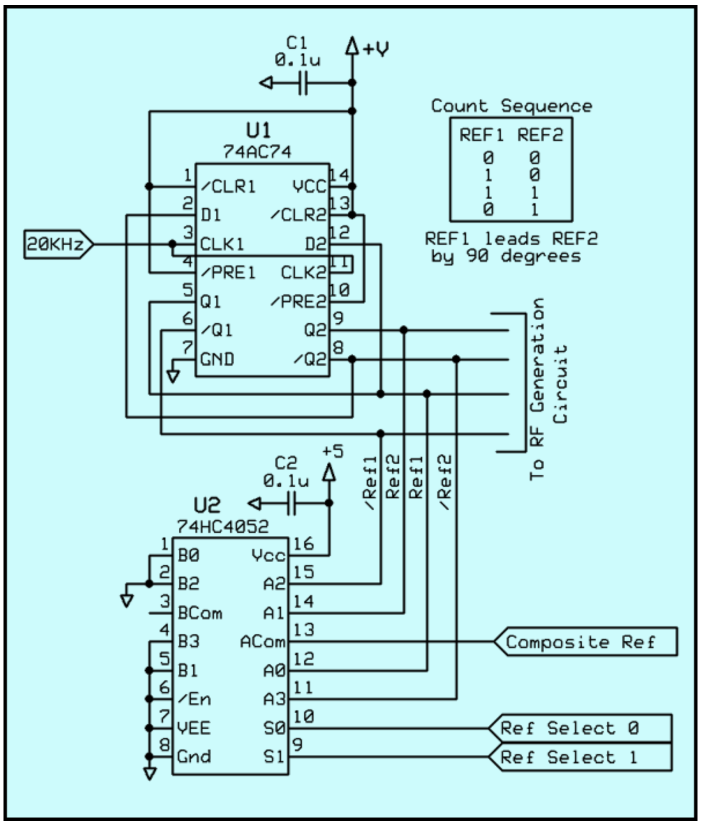

Figure 2—Twisted-Ring (Johnson) Counter, with a multiplexer to choose the desired phase for the Composite Reference. Note that it may not be necessary actually to generate the composite reference signal, since the 20 KHz control signal can be used directly to trigger ADC conversions, with the microprocessor monitoring in order to take digital readings at the proper times.

This circuit is a shift register made of two flip-flops. At each clock, the value of Q1 is shifted to Q2 and the complement of Q2 is shifted into Q1. The resulting count sequence is shown. By running through the Ref Select combinations for the multiplexer, we will get a composite reference consisting of Ref1, then Ref2, then /Ref1 and then /Ref2. The key to the phase accuracy of this circuit is that at audio frequencies, the digital propagation delays and rise/fall times are negligible compared to the period of the signal. For a 5 KHz signal, the period is 200 uS. Even if the 74AC74 had variations in propagation delay of 10 ns, this would amount to 360*(10/200,000)=0.018

degrees. And note that no matter what the propagation delay, if it is the same for each output, we still end up with perfect 90 degree spacing; it just isn't perfectly aligned with the transitions of the 20 KHz source. We don't care about that; we only care about unequal delays for different phases.

If we can make a measurement of the test signal at the positive edge of Ref1, then Ref2, then /Ref1 and then /Ref2, we will have the desired four measurements.

Depending on the RF circuitry, the circuit of Figure 3 may be a more convenient way to generate the 20 KHz quadrature-control signal from a 5 KHz reference.

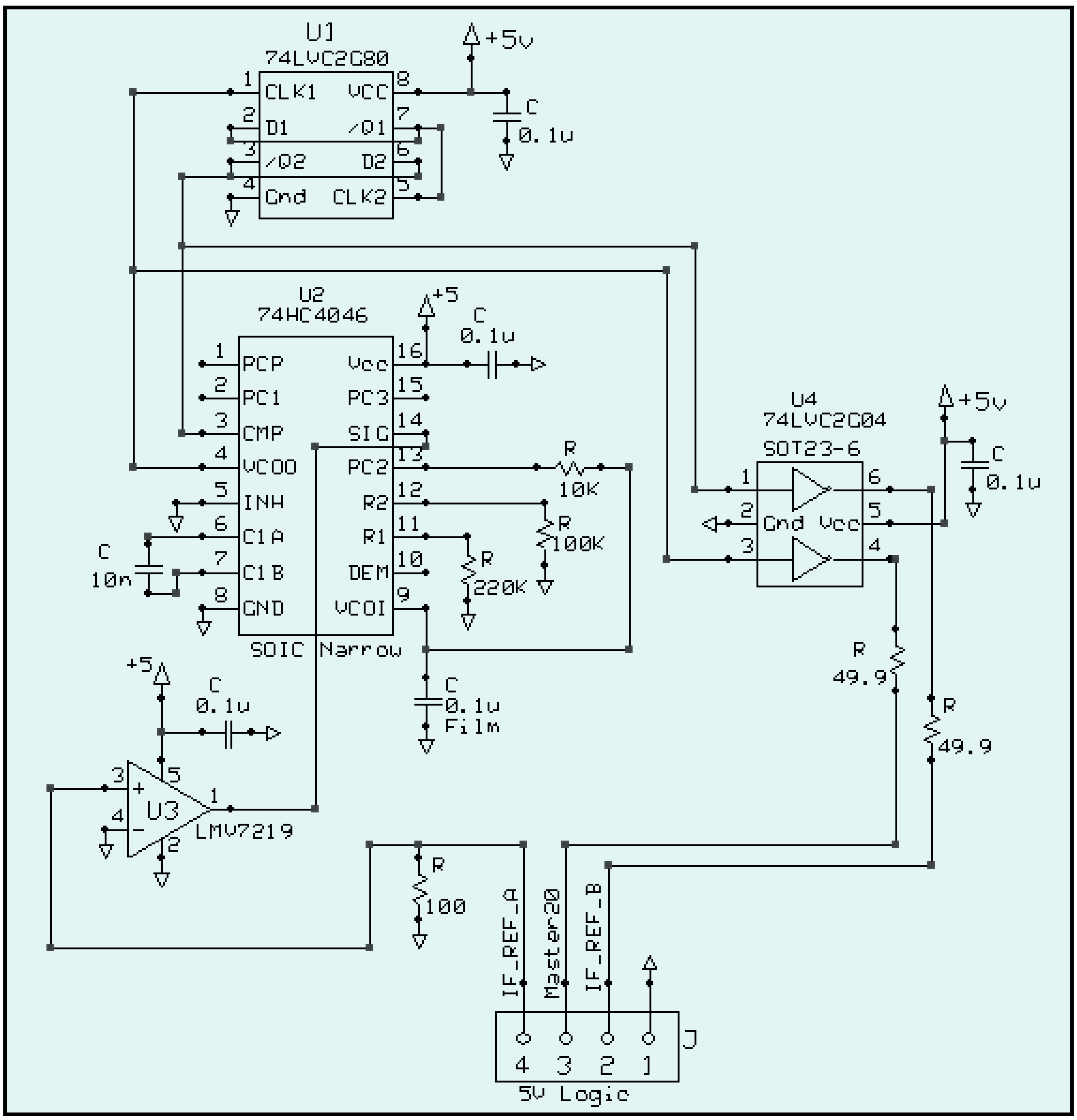

Figure 3—Alternate Signal Generation using PLL
If an IF signal reference is available from the rest of the circuitry, this

phase locked loop will multiply it by four to provide a 20KHz signal whose rising (or falling) edges can be used to trigger ADC sampling.

Assume we have an analog-to-digital converter, such as the AD7685, which will follow the value of a test signal and upon command lock its output at the then-current value of the test signal. The AD7685 has a "Conv" pin which causes a sample to be taken and conversion to commence when the Conv pin goes high. It will then digitize that sampled value in less than 3 microseconds.

Figure 4 shows the timing diagram for the four reference signals for one complete cycle, plus a little extra before and after the cycle. It also shows a composite signal made by starting with the top signal as the reference and then switching to the next lower signal at the appropriate time.

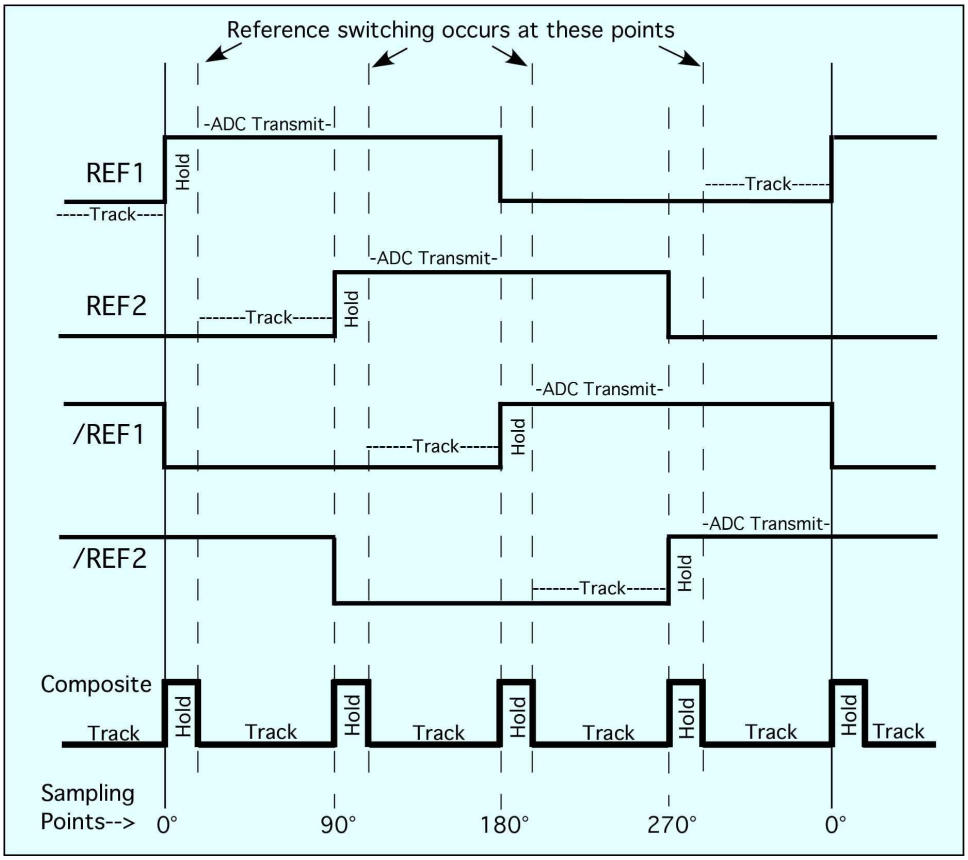

Figure 4—The four reference signals are shown. We use Ref1 until the end of its Hold period, then shift to Ref2 until the end of its Hold period, etc. The resulting composite reference is shown at the bottom, which also indicates the combined Track and Hold periods. The samples of the test signal are effectively taken at the start of each Hold period and therefore are done at 90 degree intervals.

We use the top signal (Ref1) as our reference until a short period (the Hold period) after that signal goes high. While the signal is low, our ADC will follow the value of the test signal, and when the signal goes high, the ADC will hold the value it had at the transition point. During this hold period, the ADC will be able to sample the value of the Track/Hold output. If we wait for a Hold period of 10 uS, the ADC can easily complete the conversion. If we then shift to Ref2 as the reference, the composite reference will go low, and the ADC will begin tracking again. While it is tracking it can also transmit the results to the microcontroller. If we use 5 KHz square waves for the references, each 90 degrees will represent 50 uS, so a 10 uS Hold period leaves us a 40 uS Track period, which is easily enough to re-establish accurate tracking and to complete the data transmission.

If the ADC conversion process is monitored by a microprocessor, we don't need to actually generate the four reference signals described above. The ADC conversion can be automatically triggered by transitions of the 20 KHz signal, and the processor can monitor those transitions to take readings at appropriate times.

Phase Accuracy of the IQ Method

We use our digitized measurements of I and Q to calculate the phase angle θ using the formula Arctan(I/Q). The digitization has a finite number of bits, so there will be some error in the digitized I and Q values. The question arises: How accurate must I and Q be to get an accurate estimate of θ.

If there is a certain amount of error E in the digitization process, the maximum phase error would occur if the I measurement is off by +E and Q is off by –E. By experimenting with a spreadsheet, it can also be seen that the phase error is greatest when θ=45 degrees. Finally, the greatest error occurs when the measured signal is weak, so let S equal the magnitude of the weakest signal to be measured. Thus, we can calculate the maximum resulting error in θ by the formula:

$$\text{Max error in } \theta = 45 \text{ degrees} - \text{Arctan}[\,(\sin(45)+E/S)\,/\,(\cos(45)-E/S)\,]$$

Better yet, we can use the values in Table 1, which are based on E/S expressed as a percentage. Table 1 shows that the phase error in degrees is very close to 80*E/S

IQ Error	Max Phase Error
0.25%	0.2
0.5%	0.4
1%	0.8
2%	1.6
4%	3.2

Table 1—Maximum Error (Degrees) for various percentage errors 100*E/S

Dynamic Range

With the AD7685 we have a total of 16 bits. Its "Effective Number of Bits" (ENOB) is more like 15. That means the error from rounding in the digitization process, which is half the value of the least significant bit, is 1/(2^16), or 0.000015, if we assume full scale to be 1. To account for nonlinearities, let's double that to 0.00003, which is -90.5db below the full scale reading. That means that noise, distortion and rounding errors effectively create an error equal to about -90db of the maximum signal. For strong signals, this error will be negligible as a percentage of the total signal. But for weak signals, this error can become significant. For a signal at -40db, the ratio of the error to the signal is -40 – (-90)=-50db, or about 0.3%. But for a signal at -60db, that ratio is -30db, or about 3%.

Using the percentage of digitization error for a given signal level we can look up the phase error in Table 1. The magnitude error will be a percentage equal to the percentage digitization error. Table 2 was calculated by this means. It shows the approximate errors which occur in various signal ranges, considering only errors which result from the ADC itself.

Signal Range	Max Phase Error	Max Magnitude Error
-32db to 0db	0.1 deg	0.01 db
-38db to -32db	0.2 deg	0.02 db
-44db to -38db	0.4 deg	0.04 db
-50db to -44db	0.8 deg	0.09 db
-56db to -50db	1.6 deg	0.17 db
-62db to -56db	3.2 deg	0.34 db

Table 2—approximate errors for different signal levels.
0db is assumed to be the strongest signal we can measure.

To determine dynamic range, we look down the chart to find the line with the maximum error we can accept. In many cases, the phase and magnitude accuracy of very low level signals is not as critical as for higher level signals. For example, if we measure return loss over the range 0db to -50db, one degree or 1db of error for measurements near 0db may be very significant, but the same error for measurements near -50db are virtually meaningless. Return loss of -50db implies a DUT with impedance very close to 50 ohms, and whether the deviation is 0.2 ohms or 0.3 ohms is for most purposes of no consequence, especially at high frequencies.

Therefore, we could consider this ADC to have dynamic range of 62 db, if the errors on the bottom line are acceptable for very low level signals. We can improve the accuracy/dynamic range by using a higher resolution ADC, or simply by including a processor-switchable gain boost of perhaps 30 db to be used when processing low level signals.